QCD phase transitions from relativistic hadron models.

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Abstract : The models of translationally invariant infinite nuclear matter in the relativistic mean field models are very interesting and simple, since the nucleon can connect only to a constant vector and scalar meson field. Can one connect these to the complicated phase transitions of QCD ? For an affirmative answer to this question, one must consider models where the coupling constants to the scalar and vector fields must depend on density in a non-linear way, since as such the models are not explicitly chirally invariant. Once this is ensured, indeed one can derive a quark condensate indirectly from the energy density of nuclear matter which goes to zero at large density and temperature. The change to zero condensate indicates a smooth phase transition.

1. Introduction

There is lot of interest in deriving QCD parameters from various relativistic hadronic model calculations [1], [2], [3]. In the present paper :

(1) we highlight the fact that different hadronic models, - which fit the binding energy $\mathcal{E}/\rho - M$ at the density of the infinite nuclear matter ρ_o, - also gives us the same quark condensate at ρ_o. But the expected behaviour, of $\langle \bar{q}q \rangle \to 0$ at high ρ and T, is *not built in the models*. Indeed since the models are not explicitly chirally invariant, one can combine the behaviour $\langle \bar{q}q \rangle \to 0$ with an effective nucleon mass which is not changing very much at high density. In the next section we will discuss this in more detail with respect to the recent queries [4] regarding chiral symmetry restoration.

(2) qualitative arguments are given to relate the behaviour of quark condensate $\langle \bar{q}q \rangle$, to the incompressibility of nuclear matter in hadronic models (HM, in short).

(3) we stress that to link QCD to hadronic models is important. The current attempts to derive QCD parameters from HM may provide important clues that will help in derivations of NN force from QCD. In QCD there is only one parameter Λ_{QCD} in terms of which one gets the strong coupling α_s. This in turn must control the quark and gluon condensates, although to our knowledge can only cite heuristic arguments to show this, for example in ref. [5] :

$$m_{dyn} = 3e^{1/6}\Lambda_{QCD} = 300 \ MeV \tag{1}$$

where m_{dyn} is the consituent quark mass and

$$\frac{4}{3}\pi\alpha_s\langle \bar{q}q \rangle_0 = -(m_{dyn})^3 \tag{2}$$

with $\Lambda_{QCD} = 130$ MeV.

In nuclear matter, there are more parameters, the binding energy -15.75, the equilibrium density and the incompressibility which are now leading to a specific value of $\langle \bar{q}q \rangle_\rho$, replacing them, in a sense, in terms of one parameter. The hadronic models which we consider, namely the Walecka model [6] and its modifications [7], show saturation of infinite nuclear matter through large cancellations between the attractive scalar and repulsive vector potentials. However this underlying NN force is as yet unknown from the QCD point of view, and in some way the QCD - construction of the NN force may be helped by the connection between $\langle \bar{q}q \rangle$ and the incompressibility.

The plan of the paper is as follows : in section 2 we discuss the quark condensate, how it is supposed to relate the scaling of the nucleon effective mass in a medium, the Hellmann-Feynman way to evaluate it from the quark condensate. In section 3 the nuclear models are discussed. Section 4 is devoted to results and discussions at finite density while section 5 discusses the nuclear equation of state. Section 6 contains the finite T results

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for zero baryonic density but non-zero scalar density, which we believe is and will be probed in mid-rapidity reactions with very heavy ions. Section 7 contains the summary and conclusions.

2. The quark condensate

The quark condensate in QCD is :

$$\langle \bar{q}q \rangle_0 = -(230 \pm 30 \ MeV)^3 \tag{3}$$

The non-zero value is due to the breaking of approximate chiral symmetry by the vacuum, otherwise enjoyed by the QCD Hamiltonian, by virtue of the smallness of the quark mass m_q :

$$H_{QCD} = H_0 + 2m_q\bar{q}q \tag{4}$$

the major part of the above being the chirally symmetric H_0.

A necessary but not sufficient condition for chiral symmetry restoration in QCD is that the $\langle \bar{q}q \rangle \to 0$ at high ρ and T [4]. If the symmetry is still not restored the pseudo-Goldstone particle, the pion, will be massless and one can have pion condensation [8]. Stretching the point further, can one presumably say that the effective nucleon will also be very light, if the pion becomes light ? Keeping these in mind we study the density and temperature dependence of the condensate and the corresponding sigma term. Let us begin with medium effects. Lattice studies cannot cope with finite density problems yet and QCD sumrule determination of the nucleon sigma term is not satisfactory- according to ref.[9]. In this work hadronic models of nuclear matter are employed for our purpose.

In the linear Walecka model [6], the condensate goes down linearly upto 1.5 times the nuclear matter density but then tends to *increase*, contrary to expectations based on ideas of chiral symmetry restoration. This is the same kind of result obtained by Li and Ko [3] very recently from Bonn potential in the relativistic Dirac-Brueckner approach. They also find the results unpalatable and conclude that effects like dependence of meson -nucleon coupling constants on the current quark mass and chiral invariance may become crucial in obtaining a reliable result for the density dependence of the quark condensate. Indeed we find the contrary result for variants of the Walecka model given by Zimanyi and Moszkowski (see [7]), the models are labelled by ZM and ZM3 respectively where mesons interact non-linearly and couplings are ρ-dependent. In ZM3, the effective nucleon remains massive but $\langle \bar{q}q \rangle$ goes to zero at high ρ.

Before details of our calculations we discuss a few relevant points. In the QCD sum rule approach, studied by Ioffe [10] and others [11] one gets a simple approximate expression for the nucleon mass :

$$M_N = -(8\pi^2/M^2)\langle \bar{q}q \rangle_0 \tag{5}$$

and the formula is to be evaluated for $M^2 \sim M_N{}^2$. This shows that to this order of approximation (about 10 per cent or so) the nucleon mass is controlled by the quantity in eq.(3). From this rough analysis one would conclude that at finite temperature T or density ρ one should have a scaling

$$\frac{\langle \bar{q}q \rangle_{\rho\ or\ T}}{\langle \bar{q}q \rangle_0} = [M_N{}^*/M_N]^3 \tag{6}$$

where $M_N{}^*$ is the effective nucleon mass in finite ρ or T. Indeed this was the idea put forward in [12]. Later it was suggested [1] that more careful analysis of the sum rule yields a linear scaling :

$$\frac{\langle \bar{q}q \rangle_{\rho\ or\ T}}{\langle \bar{q}q \rangle_0} = [M_N{}^*/M_N] \tag{7}$$

at least for finite density. In Figs.1a and 1b neither eq.(6) nor eq.(7) is strictly valid beyond normal nuclear matter density ρ_0 and the cubic scaling is roughly obeyed by the ZM model (Fig. 1a) while the linear one is preferred by the ZM3 model (Fig.1b). We now review how these figures can be obtained from standard nuclear matter calculations.

A concrete way of getting the quark condensate was laid down by [2] using the Hellmann-Feynman theorem :

$$\langle \psi(m_q)| \frac{d}{dm_q} H_{QCD} |\psi(m_q)\rangle = \frac{d}{dm_q} \langle \psi(m_q)|H_{QCD}|\psi(m_q)\rangle \tag{8}$$

which on using eq.(4) yields

$$2m_q(\langle \bar{q}q \rangle_\rho - \langle \bar{q}q \rangle_0) = m_q \frac{d\mathcal{E}}{dm_q}. \tag{9}$$

where the subscript ρ and 0 indicate expectation value of the relevant operator for the nucleon in nuclear matter with uniform density ρ and for the vacuum respectively. The expression is the same for finite T. $\mathcal{E}$ is the appropriate energy density. The leading term of the expression $d\mathcal{E}/dm_q$ of eq.(9) is the experimentally known quantity σ_N [2] :

$$2m_q \int d^3x (\langle N|\bar{q}q|N\rangle - \langle \bar{q}q \rangle_0) = \sigma_N = m_q \frac{dM_N}{dm_q}. \tag{10}$$

where $|N\rangle$ is the free nucleon at rest. In nuclear matter of volume V with $A = \rho V$ nucleons, translational invariance makes the quark condensate density constant and the integral in eq.(10) gives :

$$\sigma_A = 2m_q V(\langle \bar{q}q \rangle_\rho - \langle \bar{q}q \rangle_0). \tag{11}$$

Comparing eq.(11) to eq.(9) :

$$\frac{\rho}{A}\sigma_A = m_q \frac{d\mathcal{E}}{dm_q} \tag{12}$$

and (using the relation of Gell-Mann, Oakes and Renner) :

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho}{A}\frac{\sigma_A}{m_\pi^2 f_\pi^2} = 1 - \rho \frac{\sigma_{eff}}{m_\pi^2 f_\pi^2} \tag{13}$$

with $f_\pi = 93$ MeV and

$$\sigma_{eff} = \frac{\sigma_A}{A} = m_q \frac{d(\mathcal{E}/\rho)}{dm_q} = \sigma_N \frac{d(\mathcal{E}/\rho)}{dM_N} \tag{14}$$

is the effective σ-commutator for a nucleon in the nuclear medium. If we neglect $\delta\mathcal{E}$, the nucleon kinetic and interaction energy density in

$$\mathcal{E} = \rho M_N + \delta\mathcal{E} \tag{15}$$

we get $\sigma_{eff} = \sigma_N$.

Combining the eqns.(9, 11 and 15) we separate the static nucleon part and the effect of $\delta\mathcal{E}$ on σ_{eff} is :

$$\sigma_{eff} = \sigma_N - \frac{\langle \bar{q}q \rangle_\rho^{\delta\mathcal{E}}}{\langle \bar{q}q \rangle_0}\frac{m_\pi^2 f_\pi^2}{\rho}. \tag{16}$$

3. Relativistic nuclear matter models

To consider in-medium quark condensate, one needs relativistic nuclear matter [2]. Serot and Walecka [6] advocated a relativistic field theoretic description of nuclear matter based on the nucleon interacting with the scalar σ and the vector ω meson fields linearly. And this is one of the models used in ref.[2] to analyze $\langle \bar{q}q \rangle_\rho$ for $\rho \sim 1.5 \, \rho_o$ at $T = 0$. Note, however, that our results do not exactly match with theirs, since they use different values for the nuclear binding(-16 MeV and $\rho_0 = 0.17 fm^{-3}$), but are in qualitative agreement.

Spatially uniform nuclear matter, in spite of its simplicity, admits the following vital questions :

(1) What happens to $\langle \bar{q}q \rangle_\rho$ when the σ and ω fields are non-linear, inducing the couplings to be density dependent? These features were found to be very important say, in reducing the nuclear matter compressibility.

(2) What is the effect on the effective nucleon mass? Is it similar to that of $\langle \bar{q}q \rangle_\rho$? What happens at $T \neq 0$?

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Keeping these in mind we choose, apart from Walecka, two other models used in [7]. So the models are :

(1) Linear Walecka model where the coupling constants of the nucleon to σ and ω fields, g_σ and g_ω respectively, remain constant with ρ.

(2) ZM : (the usual one in the literature): It is constructed by changing the covariant derivative term in the Walecka model in such a way that, after an appropriate rescaling, the Lagrangian describes the motion of a baryon with an effective mass $M^* = m^* M$ instead of the bare mass M. This information goes to the meson-baryon coupling, modifying it to an effective scalar coupling constant, dependent on ρ, while the vector coupling constant remains the same.

(3) ZM3 : A variant of ZM is obtained if, instead of modifying the covariant derivative term, one simply modifies the kinetic energy term of the baryon. As before, after an appropriate rescaling, the Lagrangian describes a baryon of mass M^*. This information manifests not only in the scalar-baryon coupling but also in the vector-baryon coupling. Both of them now depend on density. The vector and the scalar fields are now coupled.

Connected in this way,

$$\mathcal{L}_{ZM} \equiv \mathcal{L}_{Walecka}(g_\sigma \to g_\sigma^*) \ and \ \mathcal{L}_{ZM3} \equiv \mathcal{L}_{Walecka}(g_\sigma \to g_\sigma^* \ ; \ g_\omega \to g_\omega^*)$$

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The effective coupling constants are given by : $g_\sigma^*/g_\sigma = m^*$ and $g_\omega^*/g_\omega = m^*$ where $M_N^*/M_N = m^*(\sigma) = (1 + g_\sigma\sigma/M)^{-1}$. The expression for the energy density at a given temperature T can be found in the mean field approach (MFA),

$$\mathcal{E} = \frac{g_\omega^{*2}}{2m_\omega^2}\rho_b^2 + \frac{m_\sigma^2}{2g_\sigma^{*2}}(M_N - M_N^*)^2 + \frac{\gamma}{(2\pi)^3}\int d^3k\, E^*(k)(n_k - \bar{n}_k), \tag{17}$$

where

$$\rho_b = \frac{\gamma}{(2\pi)^3}\int d^3k\,(n_k - \bar{n}_k). \tag{18}$$

Here the degeneracy factor $\gamma = 4$, n_k and $\bar{n}_k$ stand for the Fermi-Dirac distribution for baryons and antibaryons with arguments $(E^* \mp \nu)/T$ respectively. The energy $E^*(k) = (k^2 + M_N^{*2})^{\frac{1}{2}}$ and ν is an effective chemical potential which preserves the number of baryons and antibaryons in the ensemble. The effective nucleon mass M_N^*, is obtained from the minimization of $\mathcal{E}$. From eq.(9)

$$\frac{\langle\bar{q}q\rangle_{\rho,T}}{\langle\bar{q}q\rangle_0} = 1 - \frac{m_q}{m_\pi^2 f_\pi^2}\Big[\frac{\partial\mathcal{E}}{\partial M_N}\frac{\partial M_N}{\partial m_q} + \frac{\partial\mathcal{E}}{\partial m_\omega}\frac{\partial m_\omega}{\partial m_q} + \frac{\partial\mathcal{E}}{\partial m_\sigma}\frac{\partial m_\sigma}{\partial m_q} + \frac{\partial\mathcal{E}}{\partial g_\omega}\frac{\partial g_\omega}{\partial m_q} + \frac{\partial\mathcal{E}}{\partial g_\sigma}\frac{\partial g_\sigma}{\partial m_q}\Big]. \tag{19}$$

We adopt the rules [2],

$$\frac{\partial m_\sigma}{\partial m_q} = \frac{m_\sigma}{M_N}\frac{\sigma_N}{m_q}, \quad and \quad \frac{\partial m_\omega}{\partial m_q} = \frac{m_\omega}{M_N}\frac{\sigma_N}{m_q}, \tag{20}$$

and for the Walecka model the variation of the meson-couplings with m_q is unspecified and neglected. Calculating the derivatives in eq.(19), by using eq.(17), we obtain a unified expression for $\langle \bar{q}q \rangle_\rho$ at some T:

$$\frac{\langle \bar{q}q \rangle_{\rho,T}}{\langle \bar{q}q \rangle_0} = 1 - \frac{\sigma_N}{m_\pi{}^2 f_\pi{}^2} \left[\frac{m_\sigma{}^2}{g_\sigma^{*2}} (M_N - M_N^*) + (1+\alpha) \frac{m_\sigma{}^2}{g_\sigma^{*2} M_N} (M_N - M_N^*)^2 - (1+\beta) \frac{g_\omega^{*2}}{m_\omega^2 M_N} \rho_b^2 \right],$$

(21)

where, in terms of α and β, the models are describes as : Walecka $(\alpha = \beta = 0)$, ZM $(\alpha = 1 \text{ and } \beta = 0)$ and ZM3 $(\alpha = \beta = 1)$. The terms multiplied by α and β are obtained from the variation of the scalar and vector effective couplings with m_q respectively.

4. Results and Discussion for finite ρ

The parameters of the models are presented in Table I to fix the binding energy per particle to be -15.75 MeV at $\rho_0 = 0.15 fm^{-3}$. It is found that the $\langle \bar{q}q \rangle_\rho / \langle \bar{q}q \rangle_0$ and σ_{eff} are almost model independent upto a density ρ_0 (Fig. 2). Throughout this paper σ_N = 45 MeV. With this, at $\rho = \rho_0$, σ_{eff} = 44.245 MeV for all the models. As expected [2], the reduction is less than 2%. However, as evident in Fig.3, for high densies this reduction, $10 - 20\%$ for ZM models, is not negleble anymore. We stress here that $\langle \bar{q}q \rangle_{\rho_o} / \langle \bar{q}q \rangle_0 = 0.69$, for all the models. We calculate the contribution of $\delta\mathcal{E}$, using eq.(16), to the condensate and find it negligible at small densities. For example at $\rho = \rho_o$, $\langle \bar{q}q \rangle_\rho^{\delta\mathcal{E}} / \langle \bar{q}q \rangle_0 \simeq 0.005$ as compared to 0.69 for the total. Further, $\langle \bar{q}q \rangle_\rho^{\delta\mathcal{E}} \simeq -(40 MeV)^3$, reducing $\langle \bar{q}q \rangle_\rho$ to $-(203 MeV)^3$ from its vacuum value of $-(230 MeV)^3$. At this density, the kinetic and the interaction energy almost cancel each other and there is saturation. This is reflected on $\langle \bar{q}q \rangle_\rho^{\delta\mathcal{E}}$ also. The stabilizing density dependence of g_σ^* and g_ω^* make $| \langle \bar{q}q \rangle_\rho^{\delta\mathcal{E}} |$ smaller than that in the Walecka model. Thus $\langle \bar{q}q \rangle_\rho / \langle \bar{q}q \rangle_0$ reduces with density to be at par with the leading order as shown in Fig.2 for ZM models. This is more pronounced for ZM3 Model where $\langle \bar{q}q \rangle_\rho^{\delta\mathcal{E}}$ increases slowly with ρ and the corresponding σ_{eff} (Fig.3) is decreased.

If there is scaling as in eq.(6 or 7) or in any other form, the effective nucleon mass M_N^* vanishes when the condensate vanishes. This is awkward for nuclear physicists since the meaning of nuclei or nuclear matter composed of zero mass particles is ill-defined. Is there some model where the condensate will decouple ?

Indeed, as shown in Fig. 1-2, the ZM models decouple *a decreasing condensate* from the effective nucleon mass whereas in the Walecka model at $\rho > \rho_0$, $\langle \bar{q}q \rangle_\rho / \langle \bar{q}q \rangle_0$ goes up and $M_N^*/M_N \to 0$. In-medium quark condensate is governed by σ_{eff} (eq.(13)) which in turn is the derivative of $(\mathcal{E}/\rho)$ and not of M_N^* (eq.(14)). This explains why the $\langle \bar{q}q \rangle$ and M_N^* might have different density dependence and therefore the decoupling occurs. After all, at $\rho = \rho_o$, when $\mathcal{E}/\rho_o - M_N = -15.75 MeV$, $\langle \bar{q}q \rangle$ is found to be the same for all

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hadronic models with very different $M_N^*(\rho_o)$.

To investigate the role of variation of meson-couplings with m_q in the behaviour of $\langle \bar{q}q \rangle_\rho$, we have separated the contributions of these variations in Fig. 4a and 4b respectively. It is interesting that in the ZM3 model the two contributions almost cancel each other upto $\rho = 4\rho_o$ which means that in eq.(21) the term actually important is the first one

$$\frac{m_\sigma{}^2}{g_\sigma^{*2}}(M_N - M_N^*) = \frac{m_\sigma{}^2}{g_\sigma^*}\sigma \tag{22}$$

which is proportional to the sigma field. In this type of model the σ field is related not only with the scalar density (as in Walecka and ZM models) but have a new term depending on ρ, which comes from the new $\sigma - \omega$ interaction present in this model [7]. So we can conclude that the condensate goes to zero in this model only because the contributions coming from the ρ_B^2 are cancelled by the second term in eq.(21) which is proportional to

$$\frac{m_\sigma{}^2}{g_\sigma^{*2}M_N}(M_N - M_N^*)^2 = \frac{m_\sigma{}^2\sigma^2}{M_N}. \tag{23}$$

For the ZM model the variation of the scalar coupling with m_q is essential to get value of the condensate near the leading order behaviour. However, comparing the linear Walecka model with the non-linear models we use, we can conlcude that the effective density dependent coupling constants present in these models are mainly responsible for the condensate not going up at intermediate densities ($\rho/\rho_o = 1 - 2.5$) as occurs in the Walecka model.

In this context it is useful to mention that the density dependence of the coupling constants may be traced back to Brueckner Hartree Fock approaches [13]. In ZM models this dependence arises quite naturally from an interesting theoretical scaling property. It is noteworthy that in the original proposal of ZM3 [7], the authors claim that the justification for this model has to be understood in the spirit of chiral symmetry restoration. The present work confirms this, the $\langle \bar{q}q \rangle$ being restored to zero at $\rho \sim \rho_o$.

The non-linear interaction terms, in a heuristic way incorporates the effect of many-body forces leading to a suppression of the scalar field (Fig. 5). As a result, the lower bent of the nucleon mass, M_N, is checked in the medium. Moreover, in ZM3, even the vector field gets suppressed - holding back the increase of the condensate. Thus the *condensate goes down at high density* whereas the nucleon mass does not change so much. So, we have a working model of relativistic nuclear matter which has the right behaviour for $\langle \bar{q}q \rangle$ built into it. Also it gives us the hope that one may indeed construct more refined models with the same features which combines the two desirable features of vanishing $\langle \bar{q}q \rangle$ and non-vanishing effective nucleon mass.

5. Chiral Symmetry Restoration and Nuclear Matter Equation of State

Let us analyse our results in more details. From Fig. 2 we find that for the ZM3 model which is nearest to leading order, the σ_{eff} changes only 10% from σ_N at the critical density $\rho_c \sim 3.8\rho_0$. This means the contribution from interaction $\delta\mathcal{E}$ must remain small even for high densities. This can be seen clearly from the eq.(14) when it is rewritten as

$$\frac{d(\mathcal{E}/\rho)}{dM_N} = 1 - \frac{d(\delta\mathcal{E}/\rho)}{dM_N} \sim 1 - \frac{(\delta\mathcal{E}/\rho)}{M_N} . \tag{24}$$

Indeed, this is the case for ZM models as shown by the equation of state (EOS, in short) given in [7], where $\mathcal{E}/\rho - M_N$ remains closer to zero even at $\rho \sim 3-4\rho_0$. In Walecka model, however, this is not true. $\mathcal{E}/\rho$ starts rising very sharply at $\rho = 1.5\rho_0$. Obviously, the vector repulsion takes over the scalar attraction in later. In ZM3, where both fields are supressed at high density, it is no wonder that the interaction part remains subdued.

It is noteworthy that the relation given by the eq. (24) is exact for $\rho = \rho_0$. This explains the existence of the model independent point ρ_0 where $\langle \bar{q}q \rangle_\rho = 0.69$ and $\sigma_{eff} = 44.245$ MeV, keeping the binding energy per particle fixed to -15.75 MeV and $M_N = 939$ MeV.

¿From the eq.(24) we understand that σ_{eff} and $\langle \bar{q}q \rangle_\rho$ are directly connected to the EOS. Softer it is, the condensate moves closer to the leading order, realizing the restoration of chiral symmetry at high density. There is one parameter which governs the high density behaviour of the EOS for our models - that is the incompressibility, $K = 9dP/d\rho$ where P is the pressure. Not only its value at ρ_0 but also its variation with ρ should be small to obtain a softer equation of state. This is exactly what happens for ZM models, where incompressibility is small ($K(\rho_0) = 156 MeV$ for ZM3 and 220 MeV for ZM) and there is a possibility of chiral symmetry restoration. But this is impossible for Walecka model with its very stiff EOS. With these observations we can indeed connect the behaviour of the quark condensate with the nuclear matter parameters.

6. Results and Discussion for finite T

For finite temperature T, one must take into account the effect of pion gas which dominates the finite T vacuum [14], [15]. This is easily done in the models considered above using the factorization hypothesis, assuming that the pion gas is non-interacting with the nucleons at high temperature. At moderate T this may introduce errors but at high T, near the chiral restoration point surely this is not a bad approximation. For moderate T careful calculations have been done by Leutwyler and Smilga [16] using the pion-nucleon scattering data through dispersion relations and recently by Koike [17] using QCD sum rules including the effect of pion-nucleon scattering. These studies show that

the nucleon mass does not change by more than a few percent at finite T, thus supporting our effective model calculations.

In the present paper we restrict ourselves to the case of zero baryon density and finite T. This is the situation where the vector meson does not couple to the nucleon-antinucleon soup, but the scalar density is non-zero and an expression for the $\sigma_N(T)$ can be found easily. Thermal pions in the vacuum can be effectively taken through the T- dependence of f_π [17],

$$f_\pi{}^2(T) = f_\pi{}^2(1 - \frac{T^2 B_1(\frac{m_\pi}{T})}{8 f_\pi{}^2}) \tag{25}$$

where

$$B_1(z) = \frac{6}{\pi^2} \int_z^\infty dy (y^2 - z^2)^{\frac{1}{2}} \frac{1}{e^y - 1}. \tag{26}$$

The definition of the $\sigma_N(T)$ for zero chemical potential is as follows :

$$\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_\pi} = 1 - \rho_s \frac{\sigma_N(T)}{m_\pi{}^2 f_\pi{}^2(T)} \tag{27}$$

and

$$\rho_s \sigma_N(T) = [\frac{m_\sigma{}^2}{g_\sigma^{*2}}(M_N - M_N^*(T)) + (1 + \alpha)\frac{m_\sigma{}^2}{g_\sigma^{*2} M}(M_N - M_N^*(T))^2]\sigma_N(T = 0). \tag{28}$$

where

$$\rho_s = \frac{\gamma}{(2\pi)^3} \int d^3k \frac{M_N^*(T)}{E^*(k)}(n_k + \bar{n}_k)) \tag{29}$$

is the scalar density.

Using $M_N^*(T) = M_N - g_\sigma^* \sigma(T)$ and $m^* = M_N^*(T)/M_N$ in eq.(28) where the scalar field $\sigma(T) = \rho_s m^{*\alpha} g_\sigma^*/m_\sigma{}^2$ the sigma term reduces to a simple form :

$$\sigma_N(T) = (2 + \alpha - (1 + \alpha) m^*) m^{*\alpha} \sigma_N(0), \tag{30}$$

In Figs.6(a-c), the change of quark condensate with T is shown, - with and without the pion effects in the vacuum. As in the case of finite density the behaviour of the quark condensate and the M_N^* with temperature is found to be different. The condensate goes to its zero value in all the three models, - signalling perhaps, a new phase. The change in M_N^* in ZM models is very little compared to that in Walecka model. In the latter, the condensate goes down very sharply, -just like the $M_N^*(T)$. The fall off is not compensated by the rise in ρ_s : non-linearity in ZM models stabilizes $\sigma_N(T)$ to 50 MeV at critical temperature (Fig. 7) which does not happen to linear Walecka model. This is also reflected in the $\sigma_N(T)$ vs. T plot (Fig. 8). For this model $\sigma_N(T)$ increases sharply, meaning a sharp decrease in the condensate which indicates a first order phase transition

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(Fig. 6a) at $T_c \sim 180$ MeV. The ZM models, on the other hand, suggest a continuous and therefore a higher order transition. The inclusion of pions in the vacuum reduces the critical temperature (T_c) by 10%. For example in ZM3 model, T_c becomes 230 MeV from its pionless vacuum-value $T_c = 260 MeV$.

7. Summary and Conclusion

In summary it is intriguing to find that the quark condensate at normal ρ is independent of the hadron-model chosen, - models with very different M_N^* and incompressibility. But at high ρ and T it indeed depends on the modelling and it is possible to get a decoupling of the condensate from the effective nucleon mass. The fact that $\langle \bar{q}q \rangle_\rho$ decreases with density in the ZM3 model (less than linearly) shows that EOS must be soft favouring a smaller incompressibilty (less than 220 MeV). However, the decrease in $\langle \bar{q}q \rangle_\rho$ does not compensate the linear rise of the density. Consequently, σ_{eff} at high ρ reduces, by $10 - 20\%$. The reduction at ρ_0 is rather small in the hadron models we used. Further, pionic corrections arising from σ and ω fields will be there- but as commented and observed in the reference [18] - a calculation of σ_{eff} with linear sigma model - their net effect may be small.

With temperature, the condensate does go to zero. The $\sigma_N(T)$ changes more dramatically only for the Walecka model which shows a sharper increase compared to the non-linear hadron models where we have an increase of about 10% at $T = T_c$, suggesting the latter might have higher order phase transition.

References

[1] T. D. Cohen, R. J. Furnstahl, D. K. Griegel, Phys. Rev. Lett. **67**, 961, (1991).

[2] T. D. Cohen, R. J. Furnstahl, D. K. Griegel, Phys. Rev. **C45**, 1881, (1992).

[3] G. Q. Li and C. M. Ko, Phys. Lett. **338**, 118 (1994).

[4] T. Ericson, Phys. Lett. **B 321**, 312 (1994).

[5] D. Bailin, J. Cleymans, M. D. Scadron, Phys. Rev. **D31**, 164, (1985).

[6] J. D. Walecka, Ann. Phys. **83** (1974) 491; B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. Vol. 16 (Plenum, N. Y. 1986).

[7] J. Zimanyi and S. A. Moszkowski, Phys. Rev. **C44**,178 (1990) A. Delfino, C. T. Coelho and M. Malheiro, Phys. Lett. **B 245**, 361 (1995) ; A. Delfino, C. T. Coelho and M. Malheiro, Phys. Rev. **C 51**, 2188 (1995).

[8] T. Cohen and W. Broniowski, Phys. Lett. **B 342**, 25 (1995).

[9] D.K.Griegel and T. Cohen, Phys. Lett. **B 333**, 27 (1995)

[10] B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981) ; **B191**, 591(E) (1981).

[11] L. J. Reinders, H. R. Rubinstein and S. Yazaki, Phys. Rep. **127** (1985) 1.

[12] G. E. Brown and M. Rho, Phys. Rev. Lett. **66**, 2720, (1991).

[13] K. Miyazaki, Prog. Theo. Phys. **93**, 137 (1995).

[14] J. Gasser and H. Leutwyler, Phys. Lett. **184**, 83 (1987)

[15] M. Dey, V. Eletsky and B. L. Ioffe, Phys. Lett.**252**, 620 (1990).

[16] H. Leutwyler and A. V. Smilga, Nucl. Phys. **B 342**, 302 (1990).

[17] Y. Koike, Phys. Rev. **D 48**, 2315 (1993).

[18] Michael. C. Birse and Judith. A. McGovern, **MC/TH 93/07**.

Figure Captions

Fig. 1 (a-b) : Scaling laws of eq.(6) and eq. (7) respectively for Walecka and Zimanyi-Moszkowski models (ZM and ZM3).

Fig. 2 : Ratio of the condensates to the vacuum value as a function of density for three models are shown.

Fig. 3 : σ_{eff} with density.

Fig. 4 : Leading order and other contributions to condensate ratios of fig. 2 for (a) : ZM and (b) : ZM3 models.

Fig. 5 : The effective nucleon mass in the three nodels.

Fig. 6 : A plot of the ratio of the condensate with the vacuum value and the effective mass with temperature for for (a) : Walecka, (b) ZM and (c) : ZM3 models.

Fig. 7 : A plot of σ_N with scalar density for three models.

Fig. 8 : A plot of σ_N with temperature for three models.

Table 1. Coupling constants C_σ^2, C_ω^2, $m^*(\rho_o)$ and $\langle\bar{q}q\rangle_{\rho_o}/\langle\bar{q}q\rangle_0$ for different models with a fixed $\mathcal{E}/\rho_o - M_N = -15.75$ MeV at $\rho_o = 0.15 fm^{-3}$.

models	C_σ^2	C_ω^2	$m^*(\rho_o)$	$\langle\bar{q}q\rangle_{\rho_o}/\langle\bar{q}q\rangle_0$
Walecka	352.69	269.70	0.54	0.69
ZM	177.55	63.47	0.85	0.69
ZM3	440.43	303.22	0.72	0.69